METHODS USED FOR RELIABILITY ALLOCATION WITHIN COMPLEX SYSTEMS

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Abstract
The paper presents the methods used whenever the reliability of a certain system is given, to allocate the reliability in case of rubbing for a subsystem or a specific element.

Knowing that the reliability of a complex system depends both on the number of its components as well as on the reliability of each one of the components, the system reliability must be specified starting with its technical and economic study.

The system reliability is defined using the global reliability of the system that must be obtained as well as the reliability of each subsystem or element. This operation represents the allocation of the reliability.

Supposing that R* represents the specific reliability of a system and R_i* represents the reliability allocated to the i subsystem, the following relation has to be accomplished:

$$f(R_1^*, R_2^*, \dots, R_n^*) \geq R^*$$

This inequality presents the functional relation between the system reliability and the subsystems reliability. For example, if we consider a serial system, which has the following deterioration rates for its elements z(t) = λ = constant, we can write:

$$\prod R_i^* \geq R^* \text{ and } \sum \lambda_i^* \leq \left[\frac{1}{\log R^*(t)} \right] \Big/ t \qquad (1)$$

The allocation of reliability can be done using different models.

1. THE PROPORTION METHOD

Considering the deterioration rates for similar i subsystems as λ_i, the estimation of the global deterioration rate of the system is given by the relation:

$$\lambda = \sum_{i=1}^{n} \lambda_i \qquad (2)$$

The relative importance of subsystem i within the system reliability can be described as:

$$\omega_i = \frac{\lambda_i}{\sum \lambda_i} \qquad (3)$$

Then, the part of the system deterioration rate which will be allocated to the new i subsystem is defined as:

$$\lambda_i = \omega_i \lambda^* \tag{4}$$

If the subsystems operate for a t_i given period of time which is different from the t operating period of time of the whole system, there must be taken into consideration also the u_i time ratios which are called operating factors of i subsystems.

$$u_i = \frac{t_i}{t} \tag{5}$$

In that case the deterioration rate of the old system is:

$$\lambda = \sum u_i \lambda_i \tag{6}$$

The deterioration rate for the new system is:

$$\lambda^* = \sum u_i \lambda_i \tag{7}$$

To distribute proportionally it takes place the following equality:

$$\frac{u_i^* \lambda_i^*}{\lambda^*} = \frac{u_i \lambda_i}{\lambda} \tag{8}$$

It results the distribution relation for the deterioration intensities specific to subsystems:

$$\lambda_i^* = \frac{u_i}{u_i^*} \cdot \frac{\lambda^*}{\lambda} \cdot \lambda_i \tag{9}$$

2. THE MODULUS METHOD

This method deals with the decomposition of the system into modulus, characterizing the importance of each one using a coefficient ranged between 0 and 1. The value of the coefficient is 0 when the deterioration does not affect the operation of the system; the value of the coefficient is 1 when the deterioration makes the system totally unable to run; the intermediary values ranged between 0 and 1 characterize different effects that take place between the extreme points.

Thus, being given the t_i operating period of time specific to i subsystem, the t operating period of time specific to the whole system, the n_i number of modulus and the k_i subsystem importance coefficient, the subsystems allocated deterioration rate can be written as:

$$\lambda^* = \frac{n_i}{n} \cdot \frac{\lambda^*}{k_i} \cdot \frac{t}{t_i} \tag{10}$$

3. THE HARMONIC REPARTITION METHOD

For the variable deterioration rates there must be fixed a t system operating period, correlated to the specific reliability.

a) General procedure.

It is defined the upgrade coefficient of the specified R* reliability depending to R reliability of an already known system:

$$k = \frac{R*}{R} = \frac{\prod R_i^*}{\prod R_i} = \prod \frac{R_i^*}{R_i} \tag{11}$$

It is then possible to harmonic distribute the reliability, knowing that both systems contain each one n elements:

$$\frac{R_i^*}{R_i} = k^{\frac{1}{n}}; \quad k^{\frac{1}{n}} < \frac{1}{R_i}, \forall i : R_i^*(t) = R_i(t)\left[\frac{R*(t)}{R(t)}\right]^{\frac{1}{n}} \tag{12}$$

b) The limit deterioration rate procedure.

If due to technical or economic reasons it is necessary that the reliability of some subsystems not to exceed certain limits and if the use of the previous presented methods does not provide the needed reliability, the repartition has to be reconsidered in order to satisfy both the limits which are set and the specific global reliability.

Therefore the subsystems are divided in two groups: the first group contains the subsystems for which the limit R_l reliability is satisfied; the second group contains the subsystems for which the repartition has to be reconsidered.

The specific reliability of the system will be:

$$R^* = \prod_{i=1}^{n} R_i^* = \prod_{j=1}^{m} R_{L_j} \times \prod_{k=1}^{n-m} R_k^* \tag{13}$$

The repartition relation will be:

$$\prod_{k=1}^{n-m} \frac{R_k^*}{R_k} = \frac{R^*}{R} \times \prod_{j=1}^{m} \frac{R_j}{R_{L_j}} \tag{14}$$

where

$$R_k^* = R_k \times \left[\frac{R^*}{R} \times \prod_{j=1}^{m} \frac{R_j}{R_{L_j}}\right]^{\frac{1}{n-m}} \tag{15}$$

4. THE RELIABILITY-COST CRITERIA

a) The similar repartition of efforts.

The method provides for each subsystem the same relative increase of the reliability. If *R* represents the initial reliability and *R** represents the new specified reliability, the ratio below is defined as C, where C ≤ 1.

$$C = \frac{1 - R*}{1 - R} \tag{16}$$

Suppose that the systems follow an exponential distribution law.

$$C = \frac{1-e^{-\lambda^* t}}{1-e^{-\lambda t}} \cong \frac{\lambda^*}{\lambda} = \frac{m}{m^*} = \frac{MTBF}{MTBF^*} \tag{17}$$

where we can use the approximation: $e^{-\lambda t} \cong 1-\lambda t$

The *MTBF* deterioration rates multiply by a constant factor used for all subsystems; the *C* value is calculated using the following relation:

$$R^* = \prod R_i^* = \prod \left[1 - C(1-R_i)\right] \tag{18}$$

b) The similar repartition of costs.

The objectives are allocated so that the additional costs needed to reach these objectives are identical for all subsystems. Thus, the cost-reliability relation for each subsystem must be known.

In this respect, there are several proposed models, but among them the well known are:

- Darrel model:

$$C = C_0 \times \left(\frac{m}{m_0}\right)^f \tag{19}$$

where m, m_0 represent the MTBF related to C and C_0 costs.

- the exponential model:

$$\frac{1-R}{1-R_0} = e^{-a(C-C_0)^b} \tag{20}$$

$$C - C_0 = \left[\frac{1}{a}\log\left(\frac{m}{m_0}\right)\right]^b \tag{21}$$

where a, b and f are constants.

Bibliografy

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